

II. Sampling and Experimentation: Planning and conducting a study (10%-15%)

Data must be collected according to a well-developed plan if valid information on a conjecture is to be obtained. This plan includes clarifying the question and deciding upon a method of data collection and analysis.

1. Overview of methods of data collection
 1. Census - a process by which every member of a population is studied.
 2. Sample survey -
 3. Experiment
 4. Observational study
2. Planning and conducting surveys
 1. Characteristics of a well-designed and well-conducted survey
 2. Populations, samples, and random selection
 3. Sources of bias in sampling and surveys
 4. Sampling methods, including simple random sampling, stratified random sampling, and cluster sampling
3. Planning and conducting experiments
 1. Characteristics of a well-designed and well-conducted experiment
 2. Treatments, control groups, experimental units, random assignments, and replication
 3. Sources of bias and confounding, including placebo effect and blinding
 4. Completely randomized design
 5. Randomized block design, including matched pairs design

SAMPLING

I. Vocabulary

Sampling Frame - the list of all members of a population from which the sample is taken.

Representative Sample - a sample that has the essential characteristics of a population being studied and is free of any type of systematic bias.

Random Sample - each member of the population is equally likely to be included.

Simple Random Sample (SRS) - a sample of a given size is chosen in such a way that every possible sample of that size is equally likely to be chosen.

Example: If I randomly choose two players from each team in the NFL, is this a simple random sample?

Systematic Sample - the first member of a sample is chosen by some random procedure and then the rest are chosen according to some well-defined pattern.

Stratified Random Sample - subgroups of the sample, called *strata*, appear in approximately the same proportion in the sample as they do in the population.

Cluster Sample - The population is divided into sections or "clusters." Then we randomly select an entire cluster, or clusters, and sample all members of the cluster(s).

Systematic Sampling Selection Bias - the method by which a sample is taken makes it nonrepresentative of the population. Our results will favor one outcome or another making it a poor estimator of the population parameter.

Voluntary Response Sample - people choose whether or not to participate in the survey.

Convenience Sample - a sample is taken by the easiest possible way.

Judgment Sample - an expert decides who to include in a sample to make it representative.

Size Bias - the chance of being selected for a sample is based on the size of different groups in the sample.

Undercoverage - occurs when some part of the population being sampled is excluded.

Only a small part of the population is being interviewed.

Example: A pollster conducts a telephone survey to gather opinions of the general population about welfare. Persons on welfare too poor to be able to afford a telephone are systematically excluded.

Response Bias - the responses to a survey are improperly measured.

Voluntary Response Bias - people from a self-selected sample - those who feel strongly about an issue are most likely to respond.

Non-Response Bias - The possible biases of those who choose not to respond.

Questionnaire Bias - The wording of the survey question(s) is such that it influences the answers of the respondents.

Example: Would you favor or oppose a new US space program that would send astronauts to the moon?

Would you favor or oppose US government spending billions of dollars to send astronauts to the moon?

Incorrect Response Bias - untruthful responses given by the respondents. This can come from incorrect measuring devices, a survey about a sensitive topic that compels a false response, a desire to please the interviewer that prompts a false response, etc. . .

II. Selecting a Sample

Steps in Choosing a Simple Random Sample (SRS)

1. Start with a list of all the units in the population.
2. Number the units in the list.
3. Use a random number table or generator to choose units from the numbered list, one at a time, until you have as many as you need.

Steps in Choosing a Stratified Random Sample

1. Divide the units of the sampling frame into non-overlapping subgroups.

MAKE THE UNITS IN EACH STRATA SIMILAR

MAKE THE STRATA AS DIFFERENT AS POSSIBLE

2. Take a simple random sample from each subgroup. Try to make the sample sizes in each strata proportional to the population sizes of each strata.

Benefits of Stratifying

- a. *Convenience* - It is easier to sample in smaller, more compact groups than in one large group.
- b. *Coverage* - coverage of each stratum is assured.
- c. *Precision* - stratification tends to give estimates that are closer to the value for the entire population than does an SRS.

STRATIFICATION REDUCES VARIABILITY

Steps in Choosing a Cluster Sample

1. Create a numbered list of all the clusters in your population.
2. Take a simple random sample of clusters.
3. Obtain data on each individual in each cluster in your SRS.

Steps in Choosing a Two-Cluster Sample

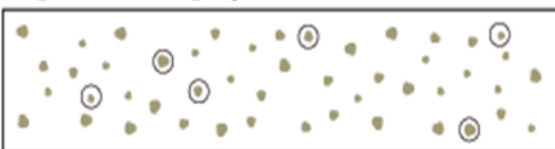
1. Create a numbered list of all the clusters in your population, and then take a simple random sample of clusters.
2. Create a numbered list of all the individuals in each cluster already selected, and then take an SRS from each cluster.

THE VARIATION WITHIN EACH CLUSTER SHOULD REFLECT THE VARIATION IN THE POPULATION

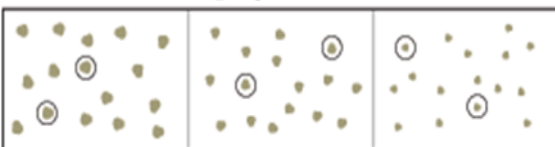
Steps in Choosing a Systematic Sample with Random Start

1. By a method such as counting off, divide your population into groups of the size you want for your sample.
2. Use a chance method to choose one of the groups for your sample.

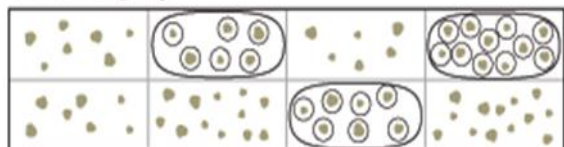
Simple Random Sampling



Stratified Random Sampling



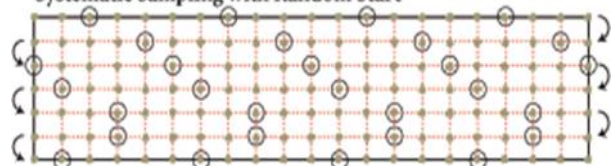
Cluster Sampling



Two-Stage Cluster Sampling



Systematic Sampling with Random Start



OBSERVATIONAL STUDIES AND EXPERIMENTS

- used to determine cause and effect.

Observational Study - no treatments get assigned to units, the conditions are already built into the units being studied.

Experiment - used to determine the extent to which treatments (explanatory variables) affects outcomes (response variables)

A well-designed experiment satisfies the principles of . . .

1. *Control* - comparing several treatments in the same environment. NOTE: Control is not synonymous with "control group". This controls for the effects of confounding variables.
2. *Randomization* - random allocation of subjects to treatment groups, not to the selection of subjects for the experiment. Randomization helps avoid bias and equalizes the effects of lurking variables among the groups.
3. *Replication* - using a large enough number of subjects to reduce chance variation in a study.

Confounding Variable - variable that has an effect on the outcome of the study but whose effects cannot be separated from those of the treatment (explanatory) variable.

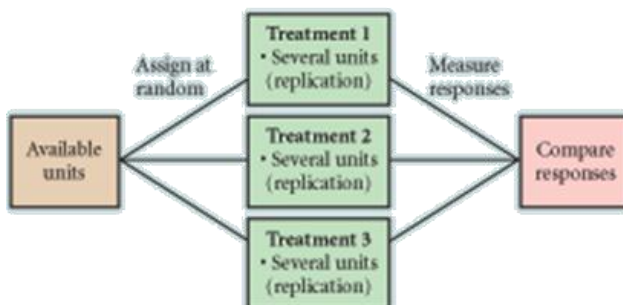
Lurking Variable - variable that has an effect on the outcomes of the study but whose influence was not part of the investigation. A lurking variable can be a confounding variable.

Placebo - a dummy treatment that separate genuine treatment effects from possible subject responses due to simply being part of an experiment. PLACEBO IS NOT NECESSARY IF A NEW TREATMENT IS COMPARED TO AN ESTABLISHED TREATMENT.

Double-blind - neither the subjects nor experimenters know which group is receiving the treatment or control.

Experimental Designs

1. Completely Randomized Design - randomly assign subjects to treatments and control groups, administration of different treatments to each group, and comparison of the outcomes.



2. Block Design - Sort your subjects into similar units. That is, they are similar because you expect them to have similar responses to the treatments. Randomly assign which unit in each block receives each treatment.

REMEMBER, EACH BLOCK GETS ALL OF THE TREATMENTS

USED TO REDUCE WITHIN-TREATMENT VARIABILITY (CONFOUNDING VARIABLES)

- a. *Matched Pairs* - Pair subjects into blocks by some characteristic or measurement (i.e. height, race, age, resting heart rate, cholesterol levels)
- b. *Matched Pairs (Repeated Measures)* - Each subject is a block in which the experiment is conducted. Each subject receives each treatment in a random order.
- c. *Randomized Block Design* - Sort your subjects into groups (blocks) of similar units. Randomly assign a treatment to each SUBJECT in each block. Try to have the same number of units assigned to each treatment within each block.

1. Data were collected in 20 cities on the percentage of women in the workforce. Data were collected in 1990 and again in 1994. Gains, or losses, in this percentage were the measurement upon which the studies conclusions were to be based. What kind of design was this?
 - I. A matched pairs design
 - II. An observational study
 - III. An experiment using a block design
 - (a) I only
 - (b) II only
 - (c) III only
 - (d) I and III only
 - (e) I and II only

2. You want to do a survey of members of the senior class at your school and want to select a *simple random sample*. You intend to include 40 students in your sample. Which of the following approaches will generate a simple random sample?
 - (a) Write the name of each student in the senior class on a slip of paper and put the papers in a container. Then randomly select 40 slips of paper from the container.
 - (b) Assuming that students are randomly assigned to classes, select two classes at random and include those students in your sample.
 - (c) From a list of all seniors, select one of the first 10 names at random. Then select every n th name on the list until you have 40 people selected.
 - (d) Select the first 40 seniors to pass through the cafeteria door at lunch.
 - (e) Randomly select 10 students from each of the four senior calculus classes.

3. Which of the following is (are) important in designing an experiment?
 - I. Control of all variables that might have an influence on the response variable.
 - II. Randomization of subjects to treatment groups.
 - III. Use of a large number of subjects to control for small-sample variability.
 - (a) I only
 - (b) I and II only
 - (c) II and III only
 - (d) I, II, and III
 - (e) II only

4. Your company has developed a new treatment for acne. You think men and women might react differently to the medication, so you separate them into two groups. Then the men are randomly assigned to two groups and the women are randomly assigned to two groups. One of the two groups is given the medication and the other is given a placebo. The basic design of this study is
 - (a) completely randomized
 - (b) blocked by gender
 - (c) completely randomized, blocked by gender
 - (d) randomized, blocked by gender and type of medication
 - (e) a matched pairs design

5. A *double-blind* design is important in an experiment because
- There is a natural tendency for subjects in an experiment to want to please the researcher.
 - It helps control for the placebo effect.
 - Evaluators of the responses in a study can influence the outcomes if they know which subjects are in the treatment group and which are in the control group.
 - Subjects in a study might react differently if they knew they were receiving an active treatment or a placebo.
 - All of the above are reasons why an experiment should be *double-blind*.
7. Which of the following is true of an experiment but not of an observational study?
- A cause-and-effect relationship can be more easily inferred.
 - The cost of conducting it is excessive.
 - More advanced statistics are needed for analysis after the data are gathered.
 - By law, the subjects need to be informed that they are part of a study.
 - Possible confounding variables are more difficult to control.
8. A study showed that persons who ate two carrots a day had significantly better eyesight than those who ate less than one carrot a week. Which of the following statements is (are) correct?
- This study provides evidence that eating carrots contributes to better eyesight.
 - The general health consciousness of people who eat carrots could be a confounding variable.
 - This is an observational study and not an experiment.
- I only
 - III only
 - I and II only
 - II and III only
 - I, II, and III
9. Which of the following situations is a cluster sample?
- Survey five friends concerning their opinions of the local hockey team.
 - Take a random sample of five voting precincts in a large metropolitan area and do an exit poll at each voting site.
 - Measure the length of time each fifth person entering a restaurant has to wait to be seated.
 - From a list of all students in your school, randomly select 20 to answer a survey about Internet use.
 - Identify four different ethnic groups at your school. From each group, choose enough respondents so that the final sample contains roughly the same proportion of each group as the school population.

Free Response

1. You are interested in the extent to which ingesting vitamin C inhibits getting a cold. You identify 300 volunteers, 150 of whom have been taking more than 1000 mg of vitamin C a day for the past month, and 150 of whom have not taken vitamin C at all during the past month. You record the number of colds during the following month for each group and find that the vitamin C group had significantly fewer colds. Is this an experiment or an observational study? Explain. What do we mean in this case when we say that the finding was *significant*?
2. Design an experiment that employs a *completely randomized design* to study the question of whether or not taking large doses of vitamin C is effective in reducing the number of colds.
3. A survey of physicians found that some doctors gave a placebo rather than an actual medication to patients who experience pain symptoms for which no physical reason can be found. If the pain symptoms were reduced, the doctors concluded that there was no real physical basis for the complaints. Do the doctors understand *the placebo effect*? Explain.
4. Explain how you would use a table of random digits to help obtain a systematic sample of 10% of the names on an alphabetical list of voters in a community. Is this a random sample? Is it a simple random sample?
5. The *Literary Digest Magazine*, in 1936, predicted that Alf Landon would defeat Franklin Roosevelt in the presidential election that year. The prediction was based on questionnaires mailed to 10 million of its subscribers and to names drawn from other public lists. Those receiving the questionnaires were encouraged to mail back their ballot preference. The prediction was off by 19 percentage points. The magazine received back some 2.3 million ballots from the 10 million sent out. What are some of the things that might have caused the magazine to be so wrong (the same techniques had produced accurate predictions for several previous elections)? (Hint: Think about what was going on in the world in 1936.)
6. Interviewers, after the 9/11 attacks, asked a group of Arab Americans if they trust the administration to make efforts to counter anti-Arab activities. If the interviewer was of Arab descent, 42% responded “yes” and if the interviewer was of non-Arab descent, 55% responded “yes.” What seems to be going on here?
7. There are three classes of statistics at your school, each with 30 students. You want to select a simple random sample of 15 students from the 90 students as part of an opinion-gathering project for your social studies class. Describe a procedure for doing this.
9. A shopping mall wants to know about the attitudes of all shoppers who visit the mall. On a Wednesday morning, the mall places 10 interviewers at a variety of places in the mall and asks questions of shoppers as they pass by. Comment on any bias that might be inherent in this approach.
12. You are going to study the effectiveness of three different SAT preparation courses. You obtain 60 high school juniors as volunteers to participate in your study. You want to assign each of the 60 students, at random, to one of the three programs. Describe a procedure for assigning students to the programs if
 - (a) you want there to be an equal number of students taking each course.
 - (b) you want each student to be assigned independently to a group. That is, each student should have the same probability of being in any of the three groups.

Multiple Choice

1. The correct answer is (e). The data are paired because there are two measurements on each city so the data are not independent. There is no treatment being applied, so this is an observational study. Matched pairs is one type of block design, but this is NOT an experiment, so III is false.
2. The answer is (a). In order for this to be an SRS, all samples of size 40 must be equally likely. None of the other choices does this [and choice (d) isn't even random]. Note that (a), (b), and (c) are probability samples.
3. The correct answer is (d). These three items represent the three essential parts of an experiment: control, randomization, and replication.
4. The correct answer is (b). You block men and women into different groups because you are concerned that differential reactions to the medication may confound the results. It is not completely randomized because it is blocked.
5. The correct answer is (e).
6. The correct answer is (b). This is an example of a voluntary response and is likely to be biased in that those that feel strongly about the issue are most likely to respond. The other choices all rely on some probability technique to draw a sample. In addition, responses (c) and (e) meet the criteria for a simple random sample (SRS).
7. The correct answer is (a). If done properly, an experiment permits you to control the variable that might influence the results. Accordingly, you can argue that the only variable that influences the results is the treatment variable.
8. The correct answer is (d). I isn't true because this is an observational study and, thus, shows a relationship but not necessarily a cause-and-effect one.
9. The correct answer is (b). (a) is a convenience sample. (c) is a systematic sample. (d) is a simple random sample. (e) is a stratified random sample.

FREE RESPONSE

1. It's an **observational study** because the researcher didn't provide a treatment, but simply observed different outcomes from two groups with at least one different characteristic. Participants self-selected themselves into either the vitamin C group or the nonvitamin C group. To say that the finding was significant in this case means that the difference between the number of colds in the vitamin C group and in the nonvitamin C group was too great to attribute to chance—it appears that something besides random variation may have accounted for the difference.
2. Identify 300 volunteers for the study, preferably none of whom have been taking vitamin C. Randomly split the group into two groups of 150 participants each. One group can be randomly selected to receive a set dosage of vitamin C each day for a month and the other group to receive a placebo. Neither the subjects nor those who administer the medication will know which subjects received the vitamin C and which received the placebo (that is, the study should be *double blind*). During the month following the giving of pills, you can count the number of colds within each group. Your measurement of interest is the difference in the number of colds between the two groups. Also, placebo effects often diminish over time.

3. The doctors probably did not understand the placebo effect. We know that, sometimes, a real effect can occur even from a placebo. If people believe they are receiving a real treatment, they will often show a change. But without a control group, we have no way of knowing if the improvement would not have been even more significant with a real treatment. The *difference* between the placebo score and the treatment score is what is important, not one or the other.
4. If you want 10% of the names on the list, you need every 10th name for your sample. Number the first ten names on the list 0, 1, 2, . . . , 9. Pick a random place to enter the table of random digits and note the first number. The first person in your sample is the person among the first 10 on the list corresponds to the number chosen. Then pick every 10th name on the list after that name. This is a random sample to the extent that, before the first name was selected, every member of the population had an equal
5. This is an instance of *voluntary response bias*. This poll was taken during the depths of the Depression, and people felt strongly about national leadership. Those who wanted a change were more likely to respond than those who were more or less satisfied with the current administration. Also, at the height of the Depression, people who subscribed to magazines and were on public lists were more likely to be well-to-do and, hence, Republican (Landon was a Republican and Roosevelt was a Democrat).
6. Almost certainly, respondents are responding in a way they feel will please the interviewer. This is a form of response bias—in this circumstance, people may just not give a truthful answer.
7. Many different solutions are possible. One way would be to put the names of all 90 students on slips of paper and put the slips of paper into a box. Then draw out 15 slips of paper at random. The names on the paper are your sample. Another way would be to identify each student by a two-digit number 01, 02, . . . , 90 and use a table of random digits to select 15 numbers. Or you could use the `randInt` function on your calculator to select 15 numbers between 1 and 90 inclusive. What you *cannot* do, if you want it to be an SRS, is to employ a procedure that selects five students randomly from each of the three classes.
9. The study suffers from *undercoverage* of the population of interest, which was declared to be all shoppers at the mall. By restricting their interview time to a Wednesday morning, they effectively exclude most people who work. They essentially have a sample of the opinions of nonworking shoppers. There may be other problems with randomness, but without more specific information about how they gathered their sample, talking about it would only be speculation.
12. (a) Many answers are possible. One solution involves putting the names of all 60 students on slips of paper, then randomly selecting the papers. The first student goes into program 1, the next into program 2, etc. until all 60 students have been assigned.

(b) Use a random number generator to select integers from 1 to 3 (like `randInt(1, 3)`) on the TI-83/84 or use a table of random numbers assigning each of the programs a range of values (such as 1–3, 4–6, 7–9, and ignore 0). Pick any student and generate a random number from 1 to 3. The student enters the program that corresponds to the number. In this way, the probability of a student ending up in any one group is $1/3$, and the selections are independent. It would be unlikely to have the three groups come out completely even in terms of the numbers in each, but we would expect it to be close.