

**III. Anticipating Patterns: Exploring random phenomena using probability and simulation (20%-30%)**

*Probability is the tool used for anticipating what the distribution of data should look like under a given model.*

**1. Probability**

1. Interpreting probability, including long-run relative frequency interpretation
2. 'Law of Large Numbers' concept
3. Addition rule, multiplication rule, conditional probability, and independence
4. Discrete random variables and their probability distributions, including binomial and geometric
5. Simulation of random behavior and probability distributions
6. Mean (expected value) and standard deviation of a random variable, and linear transformation of a random variable

**2. Combining independent random variables**

1. Notion of independence versus dependence
2. Mean and standard deviation for sums and differences of independent random variables

**3. The normal distribution**

1. Properties of the normal distribution
2. Using tables of the normal distribution
3. The normal distribution as a model for measurements

**4. Sampling distributions**

1. Sampling distribution of a sample proportion
2. Sampling distribution of a sample mean
3. Central Limit Theorem
4. Sampling distribution of a difference between two independent sample proportions
5. Sampling distribution of a difference between two independent sample means
6. Simulation of sampling distributions
7. t-distribution
8. Chi-square distribution

## I. Probability

**Sample Space (S)** - The set of all possible disjoint outcomes, or simple events, of a chance process. All of the probabilities of the outcomes in a sample space must add to 1.

ex. For the roll of a single die,  $S = \{1, 2, 3, 4, 5, 6\}$

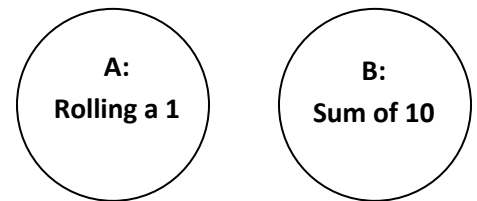
**Outcome** - one of the possible results of a chance process.

**Event** - a collection of outcomes or simple events. That is, an event is a subset of the sample space.

**Probability** - a number between 0 and 1 (0% to 100%) that tells how likely it is for an event is to happen. Probability of outcome  $A = P(A) = \frac{\# \text{ of successful outcomes}}{\text{sample space}}$

**Disjoint (Mutually Exclusive):** two different outcomes can't occur on the same opportunity.

ex. Event A: Rolling a 1. Event B: Rolling a sum of 10.



**Law of Large Numbers** - In a random sampling, the larger the sample, the closer the proportion of successes in the sample tends to be to the proportion in the population.

**Fundamental Principle of Counting** - For a multiple stage process, the number of possible outcomes for all stages taken together = the product of the total # of outcomes of each stage.

ex.

Stage	Flip a coin	Roll a 6-sided die	Roll a 3-sided die
# of outcomes	2	6	3
Total sample space for all stages = $2 \cdot 6 \cdot 3 = 36$			

### Probability of Combined Events

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cap B) = P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$P(A \text{ and } B) = P(B) \cdot P(A|B)$$

Note: If A and B are disjoint, then  $P(A \text{ and } B) = 0$

Note: If A and B are independent, then  $P(A|B) = P(A)$  and  $P(B|A) = P(B)$

### Conditional Probability

$$P(A \text{ happens given } B \text{ happens}) = P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

**Independent Events:** Events A and B are independent if and only if event A does not affect event B.

$$P(A|B) = P(A) \quad \text{OR} \quad P(B|A) = P(B)$$

Multiplication Rule for Independent Events:

$$P(A_1 \text{ and } A_2 \dots \text{ and } A_n) = P(A_1) \cdot P(A_2) \cdot \dots \cdot P(A_n) \quad \text{BECAUSE } P(A_1|A_2) = P(A_1)$$

**Probability Distribution** - gives all possible outcomes along with their probabilities.

**Expected Value:** The mean of a probability distribution =  $E(x) = \mu_x = \sum(xp)$

**Variance:**  $\text{Var}(X) = \sigma_x^2 = \sum(x - \mu_x)^2 \cdot p$       **Standard Deviation** =  $\sigma_x = \sqrt{\sigma_x^2}$

ex. Discrete probability distribution for the sum of 2 six-sided die.

<b>Sum of 2 fair die, x</b>	<b>L<sub>1</sub></b>	2	3	4	5	6	7	8	9	10	11	12
<b>Probability, p</b>	<b>L<sub>2</sub></b>	.028	.0555	.083	.111	.139	.167	.139	.111	.083	.0555	.028

NOTE: the sum of  $p = \sum p = 1$  and  $0 \leq p \leq 1$

$$E(x) = \mu_x = \sum(xp) = (2)(.028) + (3)(.0555) + \dots + 12(.028) = 7 \quad \sigma_x = 2.42$$

Calculator TIP: Find  $\mu_x$  and  $\sigma_x$  by putting x in L<sub>1</sub> and p in L<sub>2</sub>. Go to STAT → "Calc" → 1-Var Stats → List: L<sub>1</sub> and FreqList: L<sub>2</sub>

### Linear Transformation Rules

If you add c to each outcome in the probability distribution and multiply by d, what happens to  $\mu_x$  and  $\sigma_x$ ?

$$\mu_{c+dx} = c + d\mu_x \quad \text{and} \quad \sigma_{c+dx} = |d|\sigma_x$$

So if you added 3 to each outcome and multiplied each outcome by 2 in our example:

$$\mu_{c+dx} = 3 + 2(7) = 17 \quad \text{and} \quad \sigma_{c+dx} = |2|(2.42) = 2(2.42) = 4.84$$

*Notice adding 3 does NOT affect the standard deviation  $\sigma_x$ .*

### Adding and Subtracting Distributions: Mean and Standard Deviation

If you **add** two distributions x and y:       $\mu_{x+y} = \mu_x + \mu_y$        $\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$

If you **subtract** two distributions x and y:       $\mu_{x-y} = \mu_x - \mu_y$        $\sigma_{x-y} = \sqrt{\sigma_x^2 + \sigma_y^2}$

## II. Simulation: The FOUR Steps

- 1. Assumptions:** State any assumptions you are making about the situation. For example, assume the outcomes are independent.
- 2. Model:** Describe a random process to conduct one run of a simulation.
- 3. Repetition:** Run the simulation a large number of times and construct a frequency table to record the results. Record how frequently each outcome occurs.
- 4. Conclusion:** Write a conclusion in context. Remember this process is an estimate!

**AP Exam TIP:** If you're not sure how to approach a probability problem, see if you can design a simulation to get an approximate answer.

ex. The percentage of people who wash their hands after using a public restroom is 67%. Suppose you watched 4 randomly selected people using a public restroom. Use simulation to estimate the probability that all 4 washed their hands.

### III. Binomial Distribution

Recognize when a situation is BINOMIAL: 2 outcomes in each trial!

ex. Flipping a coin, washing hands or not, has a college degree or not, rolling a 4 on a die

The distribution of a random variable  $X$  (# of success) is BINOMIAL if . . .

**B:** They are *binomial* - each trial has exactly two outcomes, success and failure.

**I:** Each trial is *independent* of the others.

**N:** There is a fixed *number*,  $n$ , of trials.

**S:** The probability,  $p$ , of a *success*, is the same on each trial.

**Note:** If  $np \geq 10$  and  $n(1 - p) \geq 10$ , then a binomial distribution is approximately a normal distribution

#### Calculator Tips:

The probability of getting exactly  $X$  successes =  $\text{binompdf}(n, p, x)$

The probability of getting at most  $X$  successes =  $\text{binomcdf}(n, p, x)$

The probability of getting at least  $X$  successes =  $1 - \text{binomcdf}(n, p, x-1)$

ex. About 27% of US adults have at least a bachelor's degree. You select 100 adults at random from all adults in the US.

- a. What's the probability that exactly 30 adults have a bachelor's degree?
- b. What's the probability that at most 30 adults have a bachelor's degree?
- c. What's the probability that at least 30 adults have a bachelor's degree?

Expected Value =  $E(X) = \mu_x = np$       AND      Standard Deviation =  $\sigma_x = \sqrt{np(1-p)}$

d. How many adults do you expect to have a bachelor's degree and with what standard deviation?

#### IV. Geometric Distribution

- how many trials must you wait before the FIRST success occurs?

The distribution of a random variable  $X$  (# of success) is GEOMETRIC if ...

**B:** They are *binomial*- each trial has exactly two outcomes, success and failure.

**I:** Each trial is *independent* of the others.

**C:** The trials *continue* until the first success.

**S:** The probability,  $p$ , of a *success*, is the same on each trial.

ex. About 10% of the US population has type B blood. Suppose a technician is checking donations that may be considered independent with respect to blood type.

a. What is the probability that the first donation of type B is the 3<sup>rd</sup> one checked?

b. What is the probability that at most 3 donations will be checked before the first type B?

c. What is the probability that at least 3 donations will be checked before the first type B?

$$\text{Expected Value} = E(X) = \mu_x = \frac{1}{p} \quad \text{AND} \quad \text{Standard Deviation} = \sigma_x = \frac{\sqrt{1-p}}{p}$$

d. How many donations do you expect to check before the first type B and with what standard deviation?

#### V. Sampling Distributions

- the distribution of a summary statistic you get from taking repeated random samples.

##### Generating a Sampling Distribution

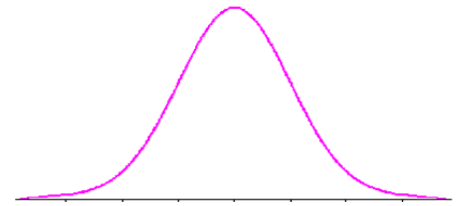
1. Take a random sample of a fixed size  $n$  from a population.
2. Compute a summary statistic (i.e. proportion  $\hat{p}$ , mean  $\bar{x}$ , etc. . . .)
3. Repeat steps 1 and 2 many times
4. Display the distribution of the summary statistic

**Standard error** - the standard deviation of the sampling distribution,  $\sigma_{\bar{x}}$ .

**Reasonably likely events** - events that lie in the middle

95% of the distribution.

**Rare (Unreasonably likely) events** - events that line in the extreme 5% of the distribution.



**Point Estimator** - a summary statistic from a sample used to estimate a parameter.

- The summary statistic should be *unbiased* which means its expected value = parameter being estimated. (i.e.  $\mu_{\bar{x}} = \mu$ )
- The summary statistic should have as little variability as possible. The standard error should decrease as the sample size increases.

**Central Limit Theorem (CLT)** - the sampling distribution becomes more normally distributed as the sample size,  $n$ , increases.

**NOTE:** If the population is approximately normal, then the sampling distribution will be approximately normal regardless of sample size.

	Shape	Center	Spread
Sampling Distribution of the <b>Sample Mean, <math>\bar{x}</math></b>	$\approx$ normal if $n \geq 30$	$\mu_{\bar{x}} = \mu$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
Sampling Distribution of the <b># of successes, <math>X</math></b>	$\approx$ normal if $np \geq 10$ and $n(1 - p) \geq 10$	$\mu_X = np$	$\sigma_X = \sqrt{np(1 - p)}$
Sampling Distribution of the <b>Sample Proportion, <math>\hat{p}</math></b>	$\approx$ normal if $np \geq 10$ and $n(1 - p) \geq 10$	$\mu_{\hat{p}} = p$	$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$

ex. The ethnicity of about 92% of the population of China is Han Chinese. Suppose you take a random sample of 1000 Chinese.

a. What is the probability of getting 90% or fewer Han Chinese in your sample?

b. What is the probability of getting a 925 or more Han Chinese?

c. What numbers of Han Chinese would be rare events? What proportions?

2.

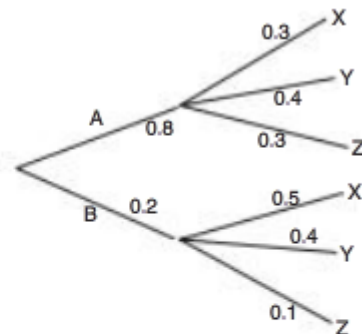
## Multiple Choice

1.

	D	E	Total
A	15	12	27
B	15	23	38
C	32	28	60
Total	62	63	125

In the table above what are  $P(A \text{ and } E)$  and  $P(C | E)$ ?

- (a) 12/125, 28/125  
 (b) 12/63, 28/60  
 (c) 12/125, 28/63  
 (d) 12/125, 28/60  
 (e) 12/63, 28/63



For the tree diagram pictured above, what is  $P(B | X)$ ?

- (a) 1/4  
 (b) 5/17  
 (c) 2/5  
 (d) 1/3  
 (e) 4/5

3. It turns out that 25 seniors at Fashionable High School took both the AP Statistics exam and the AP Spanish Language exam. The mean score on the Statistics exam for the 25 seniors was 2.4 with a standard deviation of 0.6 and the mean score on the Spanish Language exam was 2.65 with a standard deviation of 0.55. We want to combine the scores into a single score. What are the correct mean and standard deviation of the combined scores?
- (a) 5.05; 1.15  
 (b) 5.05; 1.07  
 (c) 5.05; 0.66  
 (d) 5.05; 0.81  
 (e) 5.05; you cannot determine the standard deviation from this information.
4. The GPA (grade point average) of students who take the AP Statistics exam are approximately normally distributed with a mean of 3.4 with a standard deviation of 0.3. Using Table A, what is the probability that a student selected at random from this group has a GPA lower than 3.0?
- (a) 0.0918  
 (b) 0.4082  
 (c) 0.9082  
 (d) -0.0918  
 (e) 0
5. The 2000 Census identified the ethnic breakdown of the state of California to be approximately as follows: White: 46%, Latino: 32%, Asian: 11%, Black: 7%, and Other: 4%. Assuming that these are mutually exclusive categories (this is not a realistic assumption, especially in California), what is the probability that a random selected person from the state of California is of Asian or Latino descent?
- (a) 46%  
 (b) 32%  
 (c) 11%  
 (d) 43%  
 (e) 3.5%

6. The students in problem #4 above were normally distributed with a mean GPA of 3.4 and a standard deviation of 0.3. In order to qualify for the school honor society, a student must have a GPA in the top 5% of all GPAs. Accurate to two decimal places, what is the minimum GPA Norma must have in order to qualify for the honor society?
- (a) 3.95  
 (b) 3.92  
 (c) 3.75  
 (d) 3.85  
 (e) 3.89
7. The following are the probability distributions for two random variables,  $X$  and  $Y$ :

$X$	$P(X=x)$
3	$\frac{1}{3}$
5	$\frac{1}{2}$
7	$\frac{1}{6}$

$Y$	$P(Y=y)$
1	$\frac{1}{8}$
3	$\frac{3}{8}$
4	?
5	$\frac{3}{16}$

If  $X$  and  $Y$  are independent, what is  $P(X=5 \text{ and } Y=4)$ ?

- (a)  $\frac{5}{16}$   
 (b)  $\frac{13}{16}$   
 (c)  $\frac{5}{32}$   
 (d)  $\frac{3}{32}$   
 (e)  $\frac{3}{16}$
8. The following table gives the probabilities of various outcomes for a gambling game.

Outcome	Lose \$1	Win \$1	Win \$2
Probability	0.6	0.25	0.15

What is the player's expected return on a bet of \$1?

- (a) \$0.05  
 (b) -\$0.60  
 (c) -\$0.05  
 (d) -\$0.10  
 (e) You can't answer this question since this is not a complete probability distribution.



9. You own an unusual die. Three faces are marked with the letter "X," two faces with the letter "Y," and one face with the letter "Z." What is the probability that at least one of the first two rolls is a "Y"?

(a)  $\frac{1}{6}$

(b)  $\frac{2}{3}$

(c)  $\frac{1}{3}$

(d)  $\frac{5}{9}$

(e)  $\frac{2}{9}$

10. You roll two dice. What is the probability that the sum is 6 given that one die shows a 4?

(a)  $\frac{2}{12}$

(b)  $\frac{2}{11}$

(c)  $\frac{11}{36}$

(d)  $\frac{2}{36}$

(e)  $\frac{12}{36}$

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1. A binomial event has  $n = 60$  trials. The probability of success on each trial is 0.4. Let  $X$  be the count of successes of the event during the 60 trials. What are  $\mu_x$  and  $\sigma_x$ ?

(a) 24, 3.79

(b) 24, 14.4

(c) 4.90, 3.79

(d) 4.90, 14.4

(e) 2.4, 3.79

2. Consider repeated trials of a binomial random variable. Suppose the probability of the first success occurring on the second trial is 0.25. What is the probability of success on the first trial?

(a)  $\frac{1}{4}$

(b) 1

(c)  $\frac{1}{2}$

(d)  $\frac{1}{8}$

(e)  $\frac{3}{16}$

3. To use a normal approximation to the binomial, which of the following does *not* have to be true?
- $np \geq 5$ ,  $n(1-p) \geq 5$  (or:  $np \geq 10$ ,  $n(1-p) \geq 10$ ).
  - The individual trials must be independent.
  - The sample size in the problem must be too large to permit doing the problem on a calculator.
  - For the binomial, the population size must be at least 10 times as large as the sample size.
  - All of the above are true.

4. You form a distribution of the means of all samples of size 9 drawn from an infinite population that is skewed to the left (like the scores on an easy Stats quiz!). The population from which the samples are drawn has a mean of 50 and a standard deviation of 12. Which one of the following statements is true of this distribution?
- $\mu_x = 50, \sigma_x = 12$ , the sampling distribution is skewed somewhat to the left.
  - $\mu_x = 50, \sigma_x = 4$ , the sampling distribution is skewed somewhat to the left.
  - $\mu_x = 50, \sigma_x = 12$ , the sampling distribution is approximately normal.
  - $\mu_x = 50, \sigma_x = 4$ , the sampling distribution is approximately normal.
  - $\mu_x = 50, \sigma_x = 4$ , the sample size is too small to make any statements about the shape of the sampling distribution.

5. A 12-sided die has faces numbered from 1–12. Assuming the die is fair (that is, each face is equally likely to appear each time), which of the following would give the exact probability of getting at least 10 3s out of 50 rolls?

- $\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \dots + \binom{50}{9}(0.083)^9(0.917)^{41}$ .
- $\binom{50}{11}(0.083)^{11}(0.917)^{39} + \binom{50}{12}(0.083)^{12}(0.917)^{38} + \dots + \binom{50}{50}(0.083)^{50}(0.917)^0$
- $1 - \left[ \binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \dots + \binom{50}{10}(0.083)^{10}(0.917)^{40} \right]$ .
- $1 - \left[ \binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \dots + \binom{50}{9}(0.083)^9(0.917)^{41} \right]$ .
- $\binom{50}{0}(0.083)^0(0.917)^{50} + \binom{50}{1}(0.083)^1(0.917)^{49} + \dots + \binom{50}{10}(0.083)^{10}(0.917)^{40}$ .

6. In a large population, 55% of the people get a physical examination at least once every two years. An SRS of 100 people are interviewed and the sample proportion is computed. The mean and standard deviation of the sampling distribution of the sample proportion are
- 55, 4.97
  - 0.55, 0.002
  - 55, 2
  - 0.55, 0.0497
  - The standard deviation cannot be determined from the given information.

7. Which of the following best describes the sampling distribution of a sample mean?
- It is the distribution of all possible sample means of a given size.
  - It is the particular distribution in which  $\mu_{\bar{x}} = \mu$  and  $\sigma_{\bar{x}} = \sigma$ .
  - It is a graphical representation of the means of all possible samples.
  - It is the distribution of all possible sample means from a given population.
  - It is the probability distribution for each possible sample size.
8. Which of the following is not a common characteristic of binomial and geometric experiments?
- There are exactly two possible outcomes: success or failure.
  - There is a random variable  $X$  that counts the number of successes.
  - Each trial is independent (knowledge about what has happened on previous trials gives you no information about the current trial).
  - The probability of success stays the same from trial to trial.
  - $P(\text{success}) + P(\text{failure}) = 1$ .
9. A school survey of students concerning which band to hire for the next school dance shows 70% of students in favor of hiring The Greasy Slugs. What is the approximate probability that, in a random sample of 200 students, at least 150 will favor hiring The Greasy Slugs?
- $\binom{200}{150}(0.7)^{150}(0.3)^{50}$ .
  - $\binom{200}{150}(0.3)^{150}(0.7)^{50}$ .
  - $P\left(z > \frac{150-140}{\sqrt{200(0.7)(0.3)}}\right)$ .
  - $P\left(z > \frac{150-140}{\sqrt{150(0.7)(0.3)}}\right)$ .
  - $P\left(z > \frac{140-150}{\sqrt{200(0.7)(0.3)}}\right)$ .

## Anticipating Patterns - Practice Problem SOLUTIONS

1. The correct answer is (c). There are 12 values in the A *and* E cell and this is out of the total of 125. When we are given column E, the total is 63. Of those, 28 are C.
2. The correct answer is (b).

$$P(X) = (0.8)(0.3) + (0.2)(0.5) = 0.34.$$

$$P(B | X) = \frac{(0.2)(0.5)}{(0.8)(0.3) + (0.2)(0.5)} = \frac{0.10}{0.34} = \frac{5}{17}.$$

(This problem is an example of what is known as Bayes's rule. It's still conditional probability, but sort of backwards. That is, rather than being given a path and finding the probability of going along that path— $P(X | B)$  refers to the probability of first traveling along B and then along X—we are given the outcome and asked for the probability of having gone along a certain path to get there— $P(B | X)$  refers to the probability of having gotten to X by first having traveled along B. You don't need to know Bayes's rule by name for the AP exam, but you may have to solve a problem like this one.)

3. The correct answer is (c). If you knew that the variables "Score on Statistics Exam" and "Score on Spanish Language Exam" were independent, then the standard deviation would be given by

$$\sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{(0.6)^2 + (0.55)^2} \approx 0.82.$$

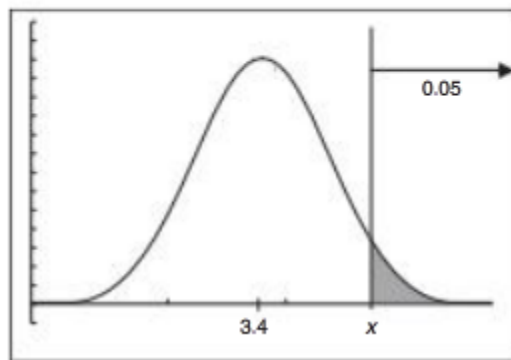
However, you cannot assume that they are independent in this situation. In fact, they aren't because we have two scores on the same people. Hence, there is not enough information.

4. The correct answer is (a).

$$P(X < 3.0) = P\left(z < \frac{3 - 3.4}{0.3} = -1.33\right) = 0.0918.$$

The calculator answer is `normalcdf(-100,3,3.4,0.3) = 0.0912`. Note that answer (d) makes no sense since probability values must be non-negative (and, of course, less than or equal to 1).

5. The correct answer is (d). Because ethnic group categories are assumed to be mutually exclusive,  $P(\text{Asian or Latino}) = P(\text{Asian}) + P(\text{Latino}) = 32\% + 11\% = 43\%$ .
6. The correct answer is (c). The situation is as pictured below:



From Table A,  $z_{.95} = 1.645$  (also, `invNorm(0.95) = 1.645`).

Hence,  $z_x = 1.645 = \frac{x - 3.4}{0.3} \Rightarrow x = 3.4 + (0.3)(1.645) = 3.89$ . Norma would need a minimum GPA of 3.89 in order to qualify for the honor society.

7. The correct answer is (c).  $P(Y=4) = 1 - \left(\frac{1}{8} + \frac{3}{8} + \frac{3}{16}\right) = \frac{5}{16}$ . Since they are independent,

$$P(X=5 \text{ and } Y=4) = P(X=5) \cdot P(Y=4) = \frac{1}{2} \cdot \frac{5}{16} = \frac{5}{32}.$$

8. The correct answer is (c). The expected value is  $(-1)(0.6) + (1)(0.25) + (2)(0.15) = -0.05$ .

9. The correct answer is (d).  $P(\text{at least one of the first two rolls is "Y"}) = P(\text{the first roll is "Y"}) + P(\text{the second roll is "Y"}) - P(\text{both rolls are "Y"}) = \frac{1}{3} + \frac{1}{3} - \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{5}{9}$ .

Alternatively,  $P(\text{at least one of the first two rolls is "Y"}) = 1 - P(\text{neither roll is "Y"}) = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$ .

10. The correct answer is (b). The possible outcomes where one die shows a 4 are highlighted in the table of all possible sums:

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>1</b>	2	3	4	<b>5</b>	6	7
<b>2</b>	3	4	5	<b>6</b>	7	8
<b>3</b>	4	5	6	7	8	9
<b>4</b>	<b>5</b>	<b>6</b>	7	<b>8</b>	<b>9</b>	<b>10</b>
<b>5</b>	6	7	8	<b>9</b>	10	11
<b>6</b>	7	8	9	<b>10</b>	11	12

There are 11 cells for which one die is a 4 (be careful not to count the **8** twice), 2 of which are 6's.

1. The correct answer is (a).

$$\mu_X = (60)(0.4) = 24\sigma_X = \sqrt{60(0.4)(0.6)} = \sqrt{14.4} = 3.79.$$

2. The correct answer is (c). If it is a binomial random variable, the probability of success,  $p$ , is the same on each trial. The probability of not succeeding on the first trial and then succeeding on the second trial is  $(1-p)p$ . Thus,  $(1-p)p = 0.25$ . Solving algebraically,  $p = 1/2$ .

3. The correct answer is (c). Although you probably wouldn't need to use a normal approximation to the binomial for small sample sizes, there is no reason (except perhaps accuracy) that you couldn't.

4. The answer is (b).

$$\mu_{\bar{x}} = \mu, \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

For small samples, the shape of the sampling distribution of  $\bar{x}$  will resemble the shape of the sampling distribution of the original population. The shape of the sampling distribution of  $\bar{x}$  is approximately normal for  $n$  sufficiently large.

5. The correct answer is (d). Because the problem stated "at least 10," we must include the term where  $x = 10$ . If the problem has said "more than 10," the correct answer would have been (b) or (c) (they are equivalent). The answer could also have been given as

$$\binom{50}{10}(0.083)^{10}(0.917)^{40} + \binom{50}{11}(0.083)^{11}(0.917)^{39} + \dots + \binom{50}{50}(0.083)^{50}(0.917)^0.$$

6. The correct answer is (d).  $\mu_{\hat{p}} = p = 0.55, \sigma_{\hat{p}} = \sqrt{\frac{(0.55)(0.45)}{100}} = 0.0497$ .

7. The correct answer is (a).

8. The correct answer is (b). This is a characteristic of a binomial experiment. The analogous characteristic for a geometric experiment is that there is a random variable  $X$  that is the number of trials needed to achieve the first success.

9. The correct answer is (c). This is actually a binomial situation. If  $X$  is the count of students "in favor," then  $X$  has  $B(200, 0.70)$ . Thus,  $P(X \geq 150) = P(X = 150) + P(X = 151) + \dots + P(X = 200)$ . Using the TI-83/84, the exact binomial answer equals 1-Binomcdf(200, 0.7, 0, 149) = 0.0695. None of the listed choices shows a sum of several binomial expressions, so we assume this is to be done as a normal approximation. We note that  $B(200, 0.7)$  can be approximated by  $N(200(0.7), \sqrt{200(0.7)(0.3)}) = N(140, 6.4807)$ . A normal approximation is OK since  $200(0.7)$  and  $200(0.3)$  are both much greater than 10. Since 75% of 200 is 150, we have  $P(X \geq 150) = P\left(z \geq \frac{150 - 140}{6.487}\right) = P(z \geq 1.543) = 0.614$ .